UR1976_09

1976/9. Determination of aquifer parameters in unconsolidated coastal sands.

W.C. Cromer

The basic aim of pump-test analysis is to establish the fundamental aquifer properties of transmissivity (*T*) and storage coefficient (*S*). Methods for determining *T* and *S* from pump-test data were developed many years ago for equilibrium and non-equilibrium radial flow conditions in confined and semi-confined aquifers. Techniques and derivations are well documented in numerous publications, on which the following comments are largely based. The reader is referred to Ferris *et al*. (1962), Bentall (1963), Jacob (1963), Lohman (1967) and Hazel (1973).

Development of separate equations suitable for analysis of data from *unconfined* aquifers has been found extremely difficult and is yet to be satisfactorily accomplished. Accordingly, confined aquifer methods are generally applied to unconfined situations. This presents a number of difficulties and spurious *T* and *S* values will be obtained if due precaution is not taken. The problems arise because confined aquifer analysis involves a number of assumptions, some of which may be invalid in unconfined aquifers. The assumptions are that:

(1) the formation is isotropic, homogeneous, of infinite areal extent and uniform saturated thickness,
(2) the piezometric surface (the water table in unconfined aquifers) prior to pumping is horizontal,
(3) the pumped bore fully penetrates the aquifer and receives water from its entire saturated thickness for the duration of pumping,
(4) water removed from storage during pumping is discharged instantaneously with concomittant head loss,
(5) storage in the bore itself is negligible,
(6) vertical flow components are negligible, and
(7) water flow during pumping is both laminar and radial.

Field experience has amply demonstrated that these assumptions do not seriously affect the usefulness of the analysis in confined aquifer conditions. But in significant respects unconfined aquifers depart unacceptably from the ideal model. Assumptions (3), (4) and (6) are most commonly violated; drawdowns are often a major fraction of the total aquifer thickness, invalidating (3); gravity drainage and associated delayed yield effects result in non-instantaneous discharge (4); and vertical flow components (6) are often significant.

Accordingly, it is necessary to apply a number of corrections to data from an unconfined aquifer prior to analysis. However, it is important to first establish from all available evidence the nature of the aquifer. The analysis of coastal sands containing thin clayey aquicludes may be more suitably carried out by means of techniques applicable to confined or semi-confined conditions. Often the aquifer will be too complex to allow a clear evaluation of *S* and *T*, and dispersion of the data is a measure of how much it departs from the ideal.

EQUILIBRIUM CONDITIONS IN UNCONFINED AQUIFERS

Initial derivation of confined aquifer analysis assumed a constraint additional to those already listed, the existence of equilibrium or near-equilibrium conditions, where continuous pumping produces a negligible change in drawdown. Subsequent refinement of the techniques has rendered this factor unnecessary, but the original equations remain applicable.

If an unconfined aquifer has been pumped for a time sufficient to produce equilibrium conditions, the *Thiem* formulae may be used to calculate T and S.

$$T = \frac{Q \log_e (r_2/r_1)^*}{2\pi (s_1 - s_2)} \qquad (1)$$

$$s = \frac{Q}{4\pi T} \cdot \log_e \frac{2.25 T t_0}{r^2 S} \qquad (2)$$

The method is time-independent, and individual drawdown measurements obtained from each of the two observation holes are sufficient to calculate T and S, assuming that the bore discharges at a constant rate. T and S may also be solved graphically by plotting s against r on semi-logarithmic paper.

Equilibrium conditions are achieved relatively quickly in confined aquifers: the cone of depression expands rapidly, and instantaneous discharge is closely approximated after relatively short pumping periods because bore yield and storage are functions of the elasticity of the aquifer and its contained water. By contrast, in fine-grained unconfined aquifers it may not be convenient or desirable to attain equilibrium. Initial discharge may be the result of elastic compression of the material and of the water, but the effect is generally minor and short-lived because of the absence of an overlying confining bed and because the water table is subjected to atmospheric pressure only. Discharge from unconfined aquifers is thus predominantly the result of gravity drainage, which proceeds at a rate determined by the vertical and horizontal permeability of the material. Since permeability is a function of mean grain size and sorting, the effect of gravity drainage varies between materials, and is greatest in fine sandy deposits. Thus, whereas the equilibrium formula may often be readily applied to data from unconfined gravel aquifers, unconsolidated sands may require days or even weeks of continuous pumping before equilibrium conditions are approximated. Clearly, this is in many cases impracticable and uneconomical.

NON-EQUILIBRIUM CONDITIONS IN UNCONFINED AQUIFERS

The development of time-dependent formulae for aquifer evaluation was a major advance in groundwater hydrology. Provided certain precautions are observed, and corrections applied, the *Theis* non-equilibrium equation for confined and semi-confined aquifers may be used in unconfined situations.

$$s = \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-u}}{u} \cdot du \qquad (3)$$

where $u = r^2 S / 4Tt$

Although equation *(3)* cannot be directly integrated, its value is given by the infinite series.

$$\int_u^\infty \frac{e^{-u}}{u} \cdot du = (-0.577216 - \log_e u + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} \cdots\cdots) \qquad (4)$$

Equation *(4)* is designated $W(u)$, 'the well function of u', and equation *(3)* is usually written:

$$s = \frac{Q}{4\pi T} \cdot W(u)$$

*Symbols are defined in Appendix 1.

Values of $W(u)$ for values of u from 10^{-15} to 9.9 are tabulated in many publications (*e.g.* Ferris *et al.*, 1962; Hazel, 1973). For any value of u, T and S are given by:

$$T = \frac{QW(u)}{4\pi s} \qquad (5)$$

and

$$S = \frac{4Ttu}{r^2} \qquad (6)$$

Because T and S cannot be obtained directly from equations *(5)* and *(6)*, Theis devised a graphical solution. Rearrangement of equations *(5)* and *(6)* shows that s is related to r^2/t in the same manner that $W(u)$ is related to u. By superimposing curves of s against r^2/t on type curves of $W(u)$ against u (both plotted on double logarithmic graph paper), values of T and S are readily obtained. The technique is fully described by Hazel (1973).

MODIFIED NON-EQUILIBRIUM EQUATION

If r is small and t large (*e.g.* for t >10 minutes, u <0.01), equation *(4)* approximates closely

$$s = \frac{Q}{4\pi T}(-0.577216 - \log_e \frac{r^2 S}{4Tt}) \qquad (7)$$

and rearranging

$$T = \frac{2 \cdot 30Q}{4\pi s}(\log_{10} \frac{2 \cdot 25Tt}{r^2 S}) \qquad (8)$$

If r is constant (*i.e.* observations are made at any one of a number of observation holes), a plot of s against t on semi-logarithmic graph paper produces a straight line of slope Δs (the drawdown per log cycle). Then,

$$T = \frac{2 \cdot 30Q}{4\pi \Delta s} \qquad (9)$$

and

$$S = \frac{2 \cdot 25Tt_0}{r^2} \qquad (10)$$

where t_0 is determined by extending the time-drawdown curve to zero drawdown.

JACOB'S (1963) CORRECTION FOR UNCONFINED AQUIFERS

For non-equilibrium conditions in thin unconfined aquifers where observed drawdowns are an appreciable fraction of the initial saturated thickness, a correction must be applied before time-drawdown curves are plotted, and before equations (*9*) and (*10*) are used. The adjusted drawdown s', is the drawdown expected in an equivalent confined aquifer.

$$s' = s - \frac{s^2}{2b} \qquad (11)$$

It is sometimes difficult to judge when the magnitude of s/b warrants adjustment, but as a safe rule, if $s^2/2b$ exceeds the accuracy of water-level measurements, Jacob's correction should be applied. The correction is important, because if s is large in relation to b, the assumption of constant aquifer thickness becomes increasingly invalid. Jacob also suggested an adjustment (a reduction of $s'S/b$) to S after drawdowns have been corrected.

$$S' = (\frac{b - s'}{b})S \qquad (12)$$

DELAYED YIELD

As discussed above, water flows by gravity drainage relatively slowly from the cone of depression in fine-grained unconfined aquifers. Formations thus exhibiting delayed yield depart from the assumption of instantaneous discharge. This can be demonstrated if *apparent* S is plotted against t during an extended pump test. The value of S will progressively increase to a constant or near-constant value. At such a stage, drainage is relatively complete and equilibrium conditions are approximated. Analysis of the data can then be attempted by using the Thiem equations *(1)* and *(2)*, their equivalent straight line solutions, or equations *(9)* and *(10)*.

NON-EQUILIBRIUM CONDITIONS IN UNCONFINED AQUIFERS EXHIBITING DELAYED YIELD

Generally, delayed yield should be assumed in fine-grained aquifers if data show that equilibrium has not been attained. Boulton (1963) devised a method of analysis for such situations. The technique also partly accounts for the effect of vertical flow components (which often invalidate analysis, especially in the vicinity of a bore during the early stages of pumping).

The time drawdown curve for the above conditions can generally be divided into three segments (Hazel, 1973).

(1) An initial segment commencing shortly after pumping starts, where the unconfined aquifer acts as a confined formation by instantaneously discharging water from storage.

(2) A middle segment showing a decreased slope due to replenishment by gravity drainage of water lost from pore spaces by instantaneous discharge. A distinct discrepancy exists between the observed data and that predicted by the Theis non-equilibrium formulae.

(3) A third segment, where gravity drainage and observed drawdowns are in equilibrium, and the drawdown curve conforms closely with the Theis type curve.

Each segment may persist for varying periods of time, depending on the characteristics of the aquifer. The effective storage coefficient is then

$$S_A + S_Y = \gamma S_A \qquad (13)$$

where S_A = volume of water instantaneously released from storage per unit drawdown per unit horizontal area (equivalent to the effective early-time storage coefficient),

S_Y = the total volume of delayed yield from storage per unit drawdown per unit horizontal area (equivalent to *specific yield*), and

$$\gamma = 1 + \frac{S_Y}{S_A}$$

Boulton's general solution to the flow equation is analogous to the Theis equation, and can be written as:

$$s = \frac{Q}{4\pi T} \cdot W(u_{AY},\ r/B) \qquad (14)$$

where $W(u_{AY},\ r/B)$ is termed 'the well function of Boulton'. In the initial stages of the pump test

$$s = \frac{Q}{4\pi T} \cdot W(u_A, r/B) \qquad (15)$$

$$\text{where } u_A = \frac{r^2 S_A}{4Tt} \qquad (16)$$

and the later stages are described by

$$s = \frac{Q}{4\pi T} \cdot W(u_Y, r/B) \qquad (17)$$

$$\text{where } u_Y = \frac{r^2 S_Y}{4Tt} \qquad (18)$$

B is the drainage factor of the aquifer

$$B = \left[\frac{T}{\alpha S_Y}\right]^{\frac{1}{2}} \qquad (19)$$

The *Boulton delay index,* α^{-1} (days), is used in conjunction with an index curve (r/B against αt) to determine the time that delayed yield ceases to affect the drawdown.

Procedures for using Boulton's method are similar to the Theis type curve method, and are fully described in a number of publications (*e.g.* Hazel, 1973). A family of type curves is constructed by plotting $W(u_{AY}, r/B)$ against $1/u_A$ and $1/u_Y$ for various values of r/B on double logarithmic graph paper. Observed data from any one of a number of observation holes are plotted as time-drawdown curves on similar paper. By matching the curves, values of $W(u_{AY}, r/B)$ and $1/u_{AY}$ are found which correspond with values of s and t. Substitution in equations *(15)*, *(16)*, *(17)* and *(18)* gives values of T_A, T_Y, S_A and S_Y. S_A values are generally in the range 10^{-5} to 10^{-2}, similar to those obtained from confined aquifers. Values of S_Y (effectively the specific yield) generally range from 10^{-2} to 0.3. The procedure is repeated for each observation hole.

The above techniques apply to time drawdown measurements made in observation bores placed at known distances from a discharging bore. Generally, the more observation holes, the better the analysis and the more reliable the results. Unconfined aquifer parameters can be obtained from discharging bores only, but the procedure is not recommended for a number of reasons. Vertical flow components are greatest in the vicinity of a pumped bore, the ratio r/b attains its highest value, and head losses due to turbulence near the pump intake render drawdown values suspect. Also, because r appears in all equations involving S, the storage coefficient cannot be obtained from discharging bores alone (although T can be determined). If no observation bores are used, it is possible to calculate S by substituting r', the effective radius of the bore, for r in the equations, but its value can only be roughly estimated. If data from observation bores are not available, it is better to use *recovery* data from the pumped bore to calculate T, since turbulent head losses are eliminated. As a corollary to the non-equilibrium formulae, Theis devised a method of analysis for recovery data from a pumped bore (Ferris *et al.*, 1962).

SITING OF OBSERVATION BORES IN UNCONSOLIDATED AQUIFERS

The siting of such bores may pose a dilemma. Not only must easily measurable drawdowns be produced, but the position of the bores relative to the discharging bore should conform to the constraints imposed by non-equilibrium analysis. Thus, to minimise vertical flow components in unconfined fine-grained aquifers, observation holes should be sited at distances greater than $2b$ from the discharging bore. Such distances may be impracticable even for medium-term pump tests. At Seven Mile Beach (Cromer, 1976), where the aquifer of medium- to fine-grained sand is 10 m thick, a pump test of 24 hours at 35 l/min^{-1} produced barely measurable drawdowns in three observation holes at 7, 10 and 15 m (all $<2b$) from the discharging bore. A subsequent test of

7 hours at 50 l/min^{-1} at a second site produced a maximum drawdown of 0.28 m in the closest of four observation holes only 2.4 m from the pumped bore. Vertical flow components at such a distance are considered to predominate over horizontal components, and the results are suspect on this basis. To conduct a satisfactory pump-test, the bore probably requires weeks of pumping, with observation holes placed at greater distances from it. In practice however, a compromise is necessary to avoid long pumping periods.

SUMMARY

In an unconfined aquifer, especially one composed of coarse sand, grit or gravel, equilibrium conditions may be readily attained and the Thiem formula may be used to analyse the data. Jacob's correction may need to be applied.

In fine-grained aquifers, instantaneous discharge rarely occurs, and delayed yield conditions probably exist. It may not be economical in terms of both time and money, to achieve equilibrium conditions. It may be desirable to compromise, by reducing the pumping period, and siting observation bores relatively close to the discharging bore. The non-equilibrium formula, the Theis recovery formula, and Boulton's method, should all be used to analyse the data in such situations (Jacob's correction may need to be applied). Each method will probably produce different results depending on the applicability of each method to the particular aquifer conditions.

One quantitative test is merely an indication of the characteristics of the aquifer, and a measure of the applicability of a particular method of analysis. Initial results may need revision on the basis of additional tests. Often a lack of knowledge of the geology of the aquifer reduces the results to a semi-quantitative category.

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[18 March 1976]

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APPENDIX 1

Symbols used

T ($m^3.day^{-1}.m^{-1}$ or $m^2.day^{-1}$): Transmissivity. The rate at which water may be transmitted through a unit strip of aquifer under unit gradient.

S (dimensionless): Storage coefficient. The volume of water which a saturated column of aquifer releases from or takes into storage per unit surface area per unit charge in head.

b (m): Saturated aquifer thickness.

Q (l/min^{-1}, $m^3.day^{-1}$): The rate of pumping of a bore.

r (m): Distance from observation bore to discharging bore.

s (m): Observed drawdown.

Δs (m): Observed drawdown per log cycle.

s' (m): Adjusted drawdown.

t (days): Time.

t_0 (days): Time, obtained by extrapolating time-drawdown curves to zero drawdown.